General Certificate of Education January 2009 Advanced Level Examination



MATHEMATICS Unit Further Pure 2

MFP2

Monday 19 January 2009 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP2.
- Answer all questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

• Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer all questions.

1 (a) Use the definitions $\sinh\theta = \frac{1}{2}(e^{\theta} - e^{-\theta})$ and $\cosh\theta = \frac{1}{2}(e^{\theta} + e^{-\theta})$ to show that

$$1 + 2\sinh^2\theta = \cosh 2\theta \tag{3 marks}$$

(b) Solve the equation

$$3\cosh 2\theta = 2\sinh \theta + 11$$

giving each of your answers in the form $\ln p$.

- (6 marks)
- 2 (a) Indicate on an Argand diagram the region for which $|z 4i| \le 2$. (4 marks)
 - (b) The complex number z satisfies $|z 4i| \le 2$. Find the range of possible values of arg z. (4 marks)
- 3 (a) Given that $f(r) = \frac{1}{4}r^2(r+1)^2$, show that

$$f(r) - f(r-1) = r^3$$
 (3 marks)

(b) Use the method of differences to show that

$$\sum_{r=n}^{2n} r^3 = \frac{3}{4}n^2(n+1)(5n+1)$$
 (5 marks)

4 It is given that α , β and γ satisfy the equations

$$\alpha + \beta + \gamma = 1$$

$$\alpha^{2} + \beta^{2} + \gamma^{2} = -5$$

$$\alpha^{3} + \beta^{3} + \gamma^{3} = -23$$

- (a) Show that $\alpha \beta + \beta \gamma + \gamma \alpha = 3$. (3 marks)
- (b) Use the identity

$$(\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha) = \alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma$$

to find the value of $\alpha\beta\gamma$.

(2 marks)

- (c) Write down a cubic equation, with integer coefficients, whose roots are α , β and γ .

 (2 marks)
- (d) Explain why this cubic equation has two non-real roots. (2 marks)
- (e) Given that α is real, find the values of α , β and γ . (4 marks)
- 5 (a) Given that $u = \cosh^2 x$, show that $\frac{du}{dx} = \sinh 2x$. (2 marks)
 - (b) Hence show that

$$\int_0^1 \frac{\sinh 2x}{1 + \cosh^4 x} \, dx = \tan^{-1}(\cosh^2 1) - \frac{\pi}{4}$$
 (5 marks)

6 Prove by induction that

$$\frac{2 \times 1}{2 \times 3} + \frac{2^2 \times 2}{3 \times 4} + \frac{2^3 \times 3}{4 \times 5} + \dots + \frac{2^n \times n}{(n+1)(n+2)} = \frac{2^{n+1}}{n+2} - 1$$

for all integers $n \ge 1$.

(7 marks)

Turn over for the next question

7 (a) Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\cosh^{-1}\frac{1}{x}\right) = \frac{-1}{x\sqrt{1-x^2}}$$
(3 marks)

(b) A curve has equation

$$y = \sqrt{1 - x^2} - \cosh^{-1}\frac{1}{x}$$
 $(0 < x < 1)$

Show that:

(i)
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{1-x^2}}{x};$$
 (4 marks)

- (ii) the length of the arc of the curve from the point where $x = \frac{1}{4}$ to the point where $x = \frac{3}{4}$ is $\ln 3$.
- **8** (a) Show that

$$(z^4 - e^{i\theta})(z^4 - e^{-i\theta}) = z^8 - 2z^4 \cos \theta + 1$$
 (2 marks)

(b) Hence solve the equation

$$z^8 - z^4 + 1 = 0$$

giving your answers in the form $e^{i\phi}$, where $-\pi < \phi \le \pi$. (6 marks)

(c) Indicate the roots on an Argand diagram. (3 marks)

END OF QUESTIONS